Pulse normalisation in optical receiver shot–noise performance*

D.J. Santos and F.J. Fraile-Peláez

Dept. Teoría de Señal y Comunicaciones, Universidad de Vigo.
ETS Ingenieros de Telecomunicación, Campus Universitario
E-36200 Vigo (Spain)

Abstract

The authors study the equivalence between two common normalisations used in the derivation of shot-noise in optical receivers. The role played by the bit rate in both approaches is discussed. They conclude that both appraches are formally equivalent, but one is conceptually preferrable. Furthermore, no absolute result can be given on the influence of bit rate on the receiver shot-noise performance. Any result is necessarily relative to the normalization used.

1 Introduction

The vast majority of optical communications systems use digital modulations in order to send information from an optical source, typically a laser diode, through an optical fiber to the optical receiver. The performance of such a communications system, as any other we can imagine, depends strongly on how well the receiver extracts the transmitted sequence from the distorted data at the end of the fiber.

Digital optical receivers typically consist of a photodetector, an amplifier, an equalizer and a device which makes a decision, based on the voltage given by the equalizer, on the bit sent. These receivers are subject to the influence of several sources of noise, the one that determines the fundamental limiting level of performance being the shot noise or quantum noise associated to the light incident on the receiver. Although of quantum-mechanical origin, there are excellent presentations of shot noise using a semiclassical model that takes into account phenomenologically the corpuscular nature of light (see e.g. [1, 2]). Personick [3, 4] was the first to study extensively the influence of shot noise in the performance of receivers in actual digital

*Manuscript of the article published in *IEE Proceedings — Optoelectronics* **146**, No. 4, pp. 201–203 (1999)

channels with inter-symbol interference distortion. Typically, one assumes that the incident optical power has the form

$$p(t) = \sum_n b_n h_p(t - nT), \tag{1}$$

where b_n are the transmitted symbols, $1/T$ is the bit rate, and $h_p(t)$ is the received optical pulse shape. Considering a deterministic linear response, represented by $h(t)$, for the cascade of the photodetector, amplifier and equalizer of the receiver (in a previous article [5] we showed that in general the impulse response of a photodetector is stochastic in nature. This randomness in the shape of the impulse response is responsible for a photonic excess noise that, in order to make contact with work of previous authors, we are neglecting in the present paper), the voltage at the output of the equalizer can be written as

$$s(t) = \sum_n s_n h_s(t - nT), \tag{2}$$

where s_n, in voltage units, contain the information associated to the symbols transmitted, and $h_s(t)$ is the shape of the equalized pulses, that we assume adimensional and satisfying the normalisation $h_s(0) = 1$.

2 Shot-noise variance

For a photodetector with a quantum efficiency η, avalanche gain $\overline{M}$, and avalanche noise-factor $F(\overline{M})$, the shot-noise variance on the decision instants is given by (see, e.g., [2, 6])

$$\sigma_s^2 = \frac{e^2 \eta F(\overline{M}) \overline{M}^2}{(2\pi)^2 h\nu} \sum_n b_n \int_{-\infty}^{\infty} d\omega H_p(\omega)\, e^{i\omega n T} H(\omega) * H(\omega), \tag{3}$$

with $H_p(\omega)$ and $H(\omega)$ Fourier transforms, respectively, of $h_p(t)$ and $h(t)$, h is Planck's constant, and ν is the optical frequency. Equation (3) is clearly a function of the transmitted sequence, b_n, so in order to obtain a conservative result for the noise, the worst case is normally studied, which corresponds to the situation in which all the symbols except the one trying to decide on are at the top level:

$$b_n = \begin{cases} b_{\max}, & n \neq 0, \\ b_0, & n = 0. \end{cases} \tag{4}$$

With this assumption, and the fact that

$$H(\omega) = \frac{s_k}{b_k} \frac{h\nu H_s(\omega)}{e\eta \overline{M} H_p(\omega)}, \tag{5}$$

equation (3) can be written as

$$\sigma_s^2 = \frac{h\nu F(\overline{M})}{\eta(2\pi)^2}[(b_0 - b_{\max})I_1' + \frac{2\pi}{T}b_{\max}\Sigma']\left(\frac{s_k}{b_k}\right)^2, \tag{6}$$

where I_1' and Σ' are given by

$$\begin{aligned} I_1' &= \int_{-\infty}^{\infty} d\omega H_p(\omega)\frac{H_s(\omega)}{H_p(\omega)} * \frac{H_s(\omega)}{H_p(\omega)} \\ \Sigma' &= \sum_n H_p(2\pi n/T)\frac{H_s(2\pi n/T)}{H_p(2\pi n/T)} * \frac{H_s(2\pi n/T)}{H_p(2\pi n/T)}. \end{aligned} \tag{7}$$

3 Normalizations

The quantities I_1' and Σ' in (6) are dependent on the shapes of the received and equalized pulses—$h_p(t)$ and $h_s(t)$, respectively—, and on the bit rate. In order to extract the latter dependence, some normalised functions are introduced. The historically first normalisation is due to Personick. He assumes the optical received pulses, that we write now as $h_{p_1}(t)$, to obey the following area-normalisation condition:

$$\int_{-\infty}^{\infty} dt\, h_{p_1}(t) = 1. \tag{8}$$

This means that the total optical *energy* incident on the photodetector, $\int_{-\infty}^{\infty} dt p(t)$, is given by $\sum_n b_n$. The b_n represent then the *energy* per symbol. Under this normalisation the following normalised functions are proposed:

$$\begin{aligned} H_p^{(1)}(\omega) &= H_{p_1}(\omega/T), \\ H_s^{(1)}(\omega) &= \frac{1}{T}H_s(\omega/T). \end{aligned} \tag{9}$$

Using these functions in (6), the following expression for the shot-noise variance is obtained:

$$\sigma_s^{2(1)} = \frac{h\nu F(\overline{M})}{\eta(2\pi)^2}[(b_0^{(1)} - b_{\max}^{(1)})I_1^{(1)} + 2\pi b_{\max}^{(1)}\Sigma^{(1)}]\left(\frac{s_k}{b_k^{(1)}}\right)^2, \tag{10}$$

where the quantities $I_1^{(1)}$ and $\Sigma^{(1)}$, given by

$$
\begin{aligned}
I_1^{(1)} &= \int_{-\infty}^{\infty} d\omega H_p^{(1)}(\omega) \frac{H_s^{(1)}(\omega)}{H_p^{(1)}(\omega)} * \frac{H_s^{(1)}(\omega)}{H_p^{(1)}(\omega)} \\
\Sigma^{(1)} &= \sum_n H_p^{(1)}(2\pi n) \frac{H_s^{(1)}(2\pi n)}{H_p^{(1)}(2\pi n)} * \frac{H_s^{(1)}(2\pi n)}{H_p^{(1)}(2\pi n)},
\end{aligned} \tag{11}
$$

are now bit-rate independent, and we have used the superscripts on the b_n symbols to distinguish their different units depending on the type of normalisation performed. Expression (10) shows that the bit rate apparently does *not* influence the shot noise.

However, a different normalisation also appears in the literature [7, 8]. In this case the received pulses, $h_{p_2}(t)$, are required to verify a different area-normalisation condition:

$$
\frac{1}{T} \int_{-\infty}^{\infty} dt \, h_{p_2}(t) = 1. \tag{12}
$$

Therefore in this case the total optical average *power* incident on the photodetector is given by $\sum_n b_n$, so the b_n, that we write now as $b_n^{(2)}$, represent the optical average *power* in the nth time slot. The normalised functions are now

$$
\begin{aligned}
H_p^{(2)}(\omega) &= \frac{1}{T} H_{p_2}(\omega/T) \\
H_s^{(2)}(\omega) &= \frac{1}{T} H_s(\omega/T).
\end{aligned} \tag{13}
$$

With this normalisation the shot-noise variance is given by

$$
\sigma_s^{2(2)} = \frac{h\nu F(\overline{M})}{\eta (2\pi)^2} [(b_0^{(2)} - b_{\max}^{(2)}) I_1^{(2)} + 2\pi b_{\max}^{(2)} \Sigma^{(2)}] \frac{1}{T} \left(\frac{s_k}{b_k^{(2)}} \right)^2, \tag{14}
$$

where the normalised integrals are also given by equations (11), except for the change of all superscripts to '(2)'. Again, both integrals, $I_1^{(2)}$ and $\Sigma^{(2)}$, are bit-rate independent. Note that under this normalisation the shot noise seems to be *proportional* to the bit rate, contrary to what has been previously stated (see equation (10)). Therefore, it does not appear clear in the current literature if the shot-noise is or not proportional to the bit rate, and this is a critical point in the practical assessment of the bit-rate influence on shot-noise receiver performance. We noticed this disagreement in a previous research paper [9], although we did not establish the comparison between both results completely. We present the proper connection between both normalisations below.

4 Comparison

The results for the shot noise expressed by equations (10) and (14) appear in the literature with a notation that does not make clear the connection between the assumptions made in both normalisations. In order to make a proper comparison between both approaches, care must be taken in understanding the units and definitions of the different functions and parameters. Specifically, it must be noted that the symbol sequences in both approaches are related by

$$b_n^{(2)} = b_n^{(1)}/T, \tag{15}$$

and that the Fourier transforms of the two families of pulses obeying, respectively, equations (8) and (12) are related by

$$H_{p_2}(\omega) = T H_{p_1}(\omega), \tag{16}$$

wich makes $H_p^{(1)}(\omega) = H_p^{(2)}(\omega)$ and therefore

$$\begin{aligned} I_1^{(2)} &= I_1^{(1)} \\ \Sigma_1^{(2)} &= \Sigma_1^{(1)}. \end{aligned} \tag{17}$$

If relations (15) and (17) are used in equations (8) and (12), it can be shown that both normalisations are equivalent, i.e.

$$\sigma_s^{2(1)} = \sigma_s^{2(2)}. \tag{18}$$

So, what is the actual dependence on bit-rate of the receiver shot-noise?

5 Discussion and conclusion

We have shown that both approaches to the study of the shot noise in digital optical receivers, although expressing an apparent contradiction in bit-rate behaviour, do in fact agree. The often cited statement 'the shot-noise power-time product is constant' (approach '(2)') cannot be formulated generally, but associated to a particular normalisation (normalisation '(2)'). Taking this statement in absolute terms may lead to misconceptions. The same applies to the statement ‘the shot-noise power-time product grows with decreasing bit-rate’ (normalisation ‘(1)’).

Phisically, the approach ‘(2)’ is based on the hypothesis that the optical average power associated to each symbol is independent of the bit rate, which seems to be a realistic assumption. However, in the approach ‘(1)’ the energy associated to each symbol is the quantity that remains constant. This fact makes the optical average power associated to each symbol depend on the transmission bit rate. We believe that this last approach is unfortunate in the sense that it involves in the normalisation hypothesis a parameter that is dependent on the transmission properties of the channel. As an illustration of this, take, for instance, a truncated exponential optical pulse shape:

$$h_p(t) = A\, e^{-t/T}, \quad 0 \leq t \leq T. \tag{19}$$

We have that $\int_{-\infty}^{\infty} dt\, h_p(t) = AT(1-1/e)$, so the normalisation (12) requires $A = 1/(1-1/e)$. However, the normalisation (8) forces the condition $A = 1/[T(1-1/e)]$ to be fulfilled, which is unrealistic, since it arbitrarily links the laser peak power to the transmitting bit rate. Therefore, although both normalisations lead to the same results, they start from phisically different grounds. In order to proceed in a consistent way, i.e. one that establishes a clear separation between transmission and reception system parameters, we encourage the use of the second approach.

References

[1] SENIOR, J.M.: ‘Optical fiber communications’ (Prentice-Hall, Hertfordshire, 1992) 2nd edn.

[2] van ETTEN, W., van der PLAATS, J.: ‘Fundamentals of optical fiber communications’ (Prentice-Hall, Hertfordshire, 1991).

[3] PERSONICK, S.D.: ‘Receiver design for digital fiber optic communication systems I’, *Bell Syst. Tech. J.*, 1973, **52**, (6), pp. 843–874.

[4] PERSONICK, S.D.: ‘Receiver design for digital fiber optic communication systems II’, *Bell Syst. Tech. J.*, 1973, **52**, (6), pp. 875–886.

[5] FRAILE-PELAEZ, F.J., SANTOS, D.J., CAPMANY, J.: ‘Photonic excess noise in a p-i-n photodetector’, *Opt. Comm.*, 1997, **135**, pp. 37–40.

[6] KEISER, G.: ‘Optical fiber communications’ (MacGraw-Hill, New York, 1991) 2nd edn.

[7] MUOI, T.V.: ‘Receiver design of optical-fiber systems’, in BASCH, E.E.: ‘Optical-fiber transmission’ (Howard Sams, Indianapolis, 1987)

[8] ALEXANDER, S.B.: ‘Optical communication receiver design’ (SPIE Optical Engineering Press, Bellingham, Washington, 1997)

[9] SANTOS, D.J., FRAILE-PELAEZ, F.J.: ‘Bit-rate influence on shot noise in digital optical receivers’, *Microwave Opt. Tech. Lett.*, 1998, **18**, (5), pp. 349–350.